

# OPTIMIZING SINGLE-PHASE TRANSFORMER DESIGN PARAMETERS USING STOCHASTIC METHODS: A PERFORMANCE EVALUATION STUDY WITH A BROAD-BASED REVIEW AND ANALYSIS

**Dr. Raju Basak**

*Professor, Techno India University, Kolkata*

## ABSTRACT

The selection of optimal design parameters for low-cost small transformers possesses significant challenges due to nonlinear objective functions and multiple constraints. Conventional optimization approach often fails to give global optima, necessitating the exploration of non-classical methods. Existing optimization schemes for single-phase transformer design parameter selection suffer from limitations, including convergence to local optima and inability to handle nonlinear objective functions, resulting in sub optimal material costs and efficiency.

The objective of this study is to design and evaluate the performance of GA, SA, and PS algorithms for minimizing material expenses in the design of a 5KVA, 230/115 volt, single-phase, core-type, dry transformer, ultimately identifying the most effective optimization approach.

The total cost of copper and iron is considered the objective function. A comparative performance evaluation of GA, SA, and PS is conducted to identify the most effective optimization scheme. The results demonstrate that non-classical techniques outperform traditional methods, yielding improved and acceptable solutions. The optimal design parameters obtained using GA, SA, and PS are analyzed and compared.

This study establishes the efficacy of stochastic optimization methods in transformer design optimization, providing valuable insights for researchers and engineers. The findings suggest that GA, SA, and PS can be effectively employed to minimize material costs and enhance transformer efficiency.

**Keywords:** *Single-phase transformer, Stochastic optimization, Genetic Algorithm, Simulated Annealing, Pattern Search, Design parameters, Material cost minimization.*

## SYMBOLS AND ABBREVIATIONS

Symbols:

1. Transformer Rating (TR)
2. Primary Voltage ( $V_p$ ), Secondary Voltage ( $V_s$ )
3. Primary Current ( $I_p$ ), Secondary Current ( $I_s$ )
4. Maximum Flux Density ( $B_m$ ), Tesla
5. Current Density ( $J$ ), A/(link unavailable)
6. Stacking Factor (SF)
7. Window Space Factor (WSF)
8. Window Dimensions: Height ( $W_h$ ), Width ( $W_w$ ), Aspect Ratio ( $W_h/W_w$ )
9. Electromotive Force per Turn (EMFt), V

10. Number of Turns: Primary ( $N_p$ ), Secondary ( $N_s$ )

11. Conductor Cross-Sectional Area: Primary (CS<sub>p</sub>), Secondary (CS<sub>s</sub>), (link unavailable)

**Abbreviations:**

1. KVA - Kilovolt-Ampere
2. C.S. - Cross-Sectional
3. EMF - Electromotive Force
4. SF - Stacking Factor
5. WSF - Window Space Factor

**Cost Parameters:**

1. Iron Cost (CI), Rs.
2. Copper Cost (CC), Rs.
3. Total Cost (TC), Rs.

## **1. INTRODUCTION**

The growing demand for efficient and cost-effective transformer designs has driven significant research efforts in optimization algorithms. The transformer industry seeks simple, effective solutions to minimize material costs, enhance performance, and reduce energy losses. Recent studies have proposed various optimization techniques, including artificial intelligence (AI) and deterministic methods.

Artificial intelligence (AI) techniques have proven instrumental in optimizing transformer design. Specifically, genetic algorithms (GAs) have successfully:

- Minimized transformer costs by optimizing material usage [12]
- Enhanced performance of cast-resin distribution transformers through efficient design configurations [13]
- Optimized toroidal core transformer designs for improved efficiency and reduced size [14]

These applications demonstrate the potential of AI-driven optimization in transformer design, paving the way for further innovations. Neural networks have facilitated the selection of winding materials. They have also enabled the prediction of transformer losses. Additionally, neural networks help in predicting transformer reactance [15, 16]. Deterministic methods, such as geometric programming, have optimized transformer design for low-frequency and high-frequency applications [17].

However, existing literature primarily focuses on minimizing specific components' costs (magnetic materials [18]) or optimizing performance parameters (output power [19], load loss [20, 21], and no-load loss [22]). Overall manufacturing cost minimization remains relatively unexplored.

Optimization is the procedure of selecting the optimal solution from a set of available alternatives. It involves systematically adjusting variables within given constraints. The goal is to minimize or maximize a specific function. This process helps in finding the best possible outcome based on the defined criteria. In real-world scenarios, there are often several feasible solutions, but optimal design refers to selecting the best one, usually under inequality constraints. Classical (deterministic) and non-classical (probabilistic) optimization tools are available. Non-classical techniques, on the basis of soft computing, have gained popularity for handling complex problems.

The present study investigates the efficacy of Pattern Search, Simulated Annealing, and Genetic Algorithm in optimizing single-phase transformer design. The primary aim is to create an optimized algorithm that reduces material costs for a 5KVA, 230/115 volt, single-phase, core-type, dry transformer, enhancing design efficiency. This study aims:

1. To compare Pattern Search, Simulated Annealing, and Genetic Algorithm performance in transformer design optimization.
2. To find the best design parameters that contribute to minimizing material costs.
3. To investigate non-classical optimization methods' applicability and limitations in transformer design.

This research contributes to existing knowledge by providing a comprehensive evaluation of non-classical optimization techniques in transformer design optimization, offering valuable insights for researchers, engineers, and industry professionals.

## **2. LITERATURE REVIEW**

The design optimization of electrical transformers has been an active area of research for several decades. This literature review provides a comprehensive overview of the developments in transformer design optimization.

### **2.1 Optimization Techniques**

The foundation for transformer design optimization was laid by **Andersen (1967) [1]** and **Kambo (1991) [10]**, who pioneered the use of mathematical programming techniques. These

studies demonstrated the effectiveness of linear and nonlinear programming in minimizing transformer costs, setting the stage for future research. **Andersen's work [1]** focused on nonlinear programming, employing a gradient-based search algorithm to optimize transformer design. The author considered various design parameters, including core geometry, winding configuration, and material selection. Andersen's study showed that mathematical optimization can significantly reduce transformer costs. Kambo's comprehensive overview [10] of mathematical programming techniques provided valuable insights into linear and nonlinear programming. The author discussed various optimization algorithms, including the simplex method and gradient-based search. Kambo's work served as a crucial resource for understanding the mathematical foundations of transformer design optimization.

Building on these foundations, **Goldberg (2011) [11]** introduced genetic algorithms (GAs) for optimization. GAs have since been widely applied in transformer design due to their ability to handle complex, nonlinear optimization problems. Goldberg's work demonstrated the effectiveness of GAs in optimizing transformer design, considering multiple objectives and constraints. The application of GAs in transformer design optimization has been further explored in subsequent studies. For example, **Amoiralis, Georgilakis and Tsili (2008) [12]** used GAs to optimize transformer design, considering minimization of copper loss, iron loss, and material costs. **Elia et al. (2006) [13]** combined GAs with simulated annealing to optimize transformer design. More recently, researchers have explored other optimization techniques, such as particle swarm optimization (PSO) and artificial neural networks (ANNs). **Jabr (2005) [17]** reviewed recent advances in transformer design optimization, highlighting the potential of evolutionary algorithms like GAs and PSO.

*The development of optimization techniques for transformer design has progressed significantly since Andersen's and Kambo's pioneering work. Mathematical programming techniques, GAs, and other evolutionary algorithms have been successfully applied to optimize transformer design, reducing costs and improving efficiency.*

## 2.2 Computer-Aided Design and Multi-Objective Optimization

The combination of computer-aided design (CAD) and multi-objective optimization has transformed transformer design. **Rubaai (1994) [3] and Amoiralis, Georgilakis and Tsili (2008) [12]** pioneered this field, showcasing the benefits of CAD and the effectiveness of genetic algorithms (GAs) in achieving optimal transformer design. Rubaai's work [3] demonstrated the potential of CAD in transformer design, highlighting benefits such as improved accuracy, reduced design time, enhanced visualization, automated optimization. Rubaai's study showed that CAD enables designers to explore complex geometries and optimize performance, paving the way for future research. **Amoiralis, Georgilakis and Tsili's (2008) [12]** research built on Rubaai's findings, introducing multi-objective optimization using GAs. The authors considered multiple objectives, including minimization of copper loss, minimization of iron loss, minimization of material costs. **Amoiralis, Georgilakis and Tsili's (2008) [12]** study demonstrated the effectiveness of GAs in achieving optimal transformer design, considering trade-offs between competing objectives.

Subsequent studies further explored the application of CAD and multi-objective optimization in transformer design. For example: **Lia et al. (2006) [13]** combined GAs with simulated annealing to optimize transformer design. **Tutkun et al. (2004) [14]** used GAs to optimize magnetic core design.

Recent research has integrated CAD with other optimization techniques, such as particle swarm optimization (PSO) and artificial neural networks (ANNs).

**Jabr (2005) [17]** reviewed recent advances in transformer design optimization, highlighting the potential of evolutionary algorithms like GAs and PSO. The integration of CAD and multi-

objective optimization has transformed transformer design, enabling Improved efficiency, reduced costs, enhanced performance, Increased reliability.

*The development of computer-aided design and multi-objective optimization techniques has significantly advanced transformer design. CAD and GAs have been successfully applied to optimize transformer design, reducing costs and improving efficiency.*

### 2.3 Magnetic Core Design and Robust Optimization

Magnetic core design and robust optimization are critical aspects of transformer design, ensuring efficient and reliable performance. **Tutkun et al. (2004) [14]** and **Geromel et al. (2002) [16]** pioneered research in these areas, presenting design optimization methods for magnetic cores and robust optimization of transformer design. Tutkun et al.'s study [14] focused on optimizing magnetic core design using genetic algorithms (GAs). The authors considered various design parameters, including core geometry, material selection, and winding configuration. Their research demonstrated the potential of GAs in optimizing magnetic core design, resulting in improved efficiency and reduced losses. **Geromel et al.'s work [16]** introduced robust optimization techniques to ensure reliable transformer performance. The authors addressed uncertainties in design parameters and load conditions, employing linear matrix inequality (LMI) approaches to optimize transformer design. Their study highlighted the importance of robust optimization in mitigating performance degradation and ensuring reliable operation. Subsequent research built upon these findings, exploring advanced optimization techniques and robust design methods. For example, **Elia et al. (2006) [13]** combined GAs with simulated annealing to optimize transformer design, while **Jabr (2005) [17]** reviewed recent advances in transformer design optimization, emphasizing the potential of evolutionary algorithms. The integration of magnetic core design optimization and robust optimization has significantly enhanced transformer design:

*The development of magnetic core design and robust optimization techniques has substantially advanced transformer design. These studies demonstrate the effectiveness of GAs and robust optimization methods in ensuring efficient and reliable transformer performance.*

### 2.4 Recent Advances and Future Directions

**Jabr's** comprehensive review (2005) [17] showcased recent breakthroughs in transformer design optimization, highlighting the potential of evolutionary algorithms like: Genetic Algorithms (GAs), Particle Swarm Optimization (PSO).

Future Research Directions

*To further propel innovation, future research should focus on the following:*

1. Sophisticated Optimization Algorithms
2. Developing hybrid algorithms combining GAs, PSO, and other techniques
3. Exploring novel optimization methods, such as swarm intelligence and ant colony optimization
4. Machine Learning Integration
5. Applying machine learning to predict optimal design parameters
6. Integrating machine learning with evolutionary algorithms
7. Multi-Objective Optimization with Robust Optimization
8. Developing robust multi-objective optimization frameworks
9. Addressing uncertainties in design parameters and load conditions
10. Advanced Materials and Technologies
11. Optimizing transformer design for emerging materials and technologies
12. Exploring new applications in renewable energy and electric vehicles
13. Industrial Collaboration and Practical Implementation
14. Collaborating with industry partners to implement optimization techniques

## 3. Objective Function Development

This study focuses on optimizing the design of a 5 KVA, 230/115 V, 50 Hz, dry-type transformer. The material cost is selected as the objective function.

### 3.1 Design Variables and Constraints

Two dynamic variables are chosen:

1. EMF constant
2. Window height/width ratio

Standard values are adopted for other design and decision variables:

- Core material: Cold Rolled Steel (CRS)
- Conductor material: Copper
- Flux-density: 1.4 wb/m<sup>2</sup>
- Current density: 2.4 A/mm<sup>2</sup>
- Core type: Stepped core with 3 steps
- Window space factor: 0.4
- Stacking factor: 0.93 (with CRS core)

### 3.2 Design Constraints

Practical considerations for standard transformers dictate the following design constraints:

1. Full-load efficiency at 0.85 lagging power factor:  $\geq 96\%$
2. Voltage regulation at full load and 0.85 lagging power factor:  $\leq 4\%$
3. No-load current:  $\leq 2\%$
4. Temperature rise at full load:  $\leq 50^{\circ}\text{C}$

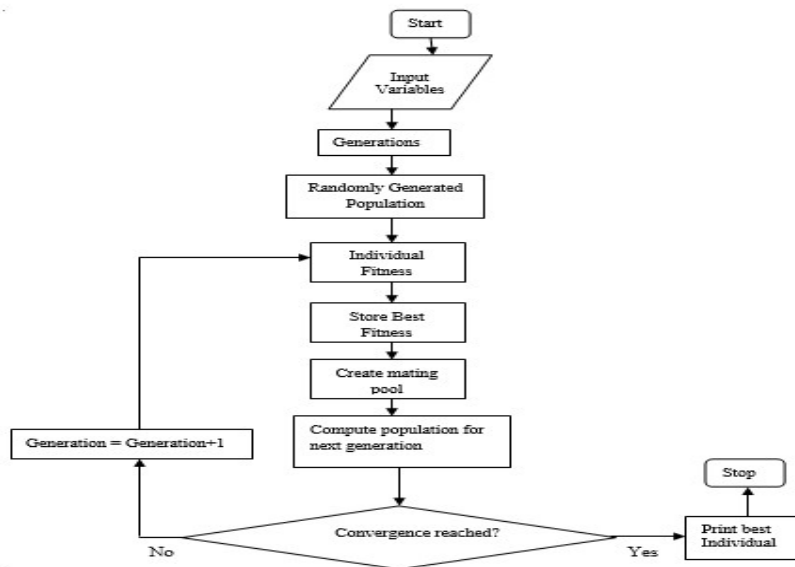
### 3.3 The Optimizing Function

The material cost (Rs. =  $CM = 1131(\sqrt{R_w K} + \sqrt{K/R_w}) + 3371K^{1.5} + 2520/\sqrt{K} + 803/(K^{1.5}\sqrt{R_w})$ )

Now we have to optimize (minimize) the function.

## 4. GENETIC ALGORITHM

Genetic algorithms were initially developed by John Holland and later advanced by Holland and DeJong. They were popularized by Goldberg. Numerous researchers, through their research activities, have expanded its applications and refined various aspects of the algorithm, making it widely applicable across nearly every field of engineering and research. By simulating natural genetic processes, genetic algorithms have become a fundamental tool for optimization. Figure 1 illustrates the optimization flow chart for the genetic algorithm.



**Fig 1. Diagrammatic Representation of Genetic Algorithm through Flow Chart**

A genetic algorithm involves three main steps: reproduction, crossover, and mutation. The process begins with an initial population of randomly encoded chromosomes, each representing a potential solution. These chromosomes are then selected for recombination using the crossover operator. The crossover operator creates improved offspring by combining genetic information from the selected chromosomes. This process continues through successive generations, gradually evolving better solutions. Probabilistic bit mutation is applied to introduce diversity, which enhances the quality of the solutions. This process of natural selection continues through a set number of generations, culminating in a final population of highly fit chromosomes that represent optimal or near-optimal solutions to the problem.

## 5. SIMULATED ANNEALING

### Simulated Annealing Algorithm

Simulated Annealing (SA) optimizes functions by mimicking metal crystallization.

#### Principles

1. Initialize with high temperature (T) and gradually decrease.
2. Calculate energy change probability (P) using Boltzmann's constant (K) and temperature (T).

#### SA Algorithm

1. Initialize:  $x$ ,  $\epsilon$ ,  $T$ ,  $n$ ,  $t=0$ .
2. Generate neighboring point  $x(t+1)$  via random perturbation  $N$ .
3. Evaluate:
  - $\Delta f \leq 0$ : accept  $x(t+1)$
  - Else: generate  $r$  (0, 1)
  - $r \leq P$ : accept  $x(t+1)$ ; otherwise, reject.
4. Check termination:
  - $|x(t+1) - x(t)| < \epsilon$  and  $T$  small: terminate
  - Else: update  $T$  via cooling schedule; repeat Step 2.

5. End.

#### Key Parameters

1. Initial temperature ( $T$ )
2. Iterations per temperature ( $n$ )
3. Cooling schedule

#### Considerations

1. High  $T$  increases convergence iterations.
2. Low  $T$  may lead to insufficient exploration.
3. High iteration count recommended.

## 6. Pattern Search Algorithm

The Pattern Search method iteratively searches for the minimum along a specified direction. This direction, called the pattern direction, is determined by two points: the starting point ( $x_0$ ) and the point obtained after univariate steps ( $x$ ).

### Hooke-Jeeves Pattern Search Technique

This technique establishes a series of search directions to find the optimal solution.

#### 6.1 Pattern Search Algorithm Steps

1. Initialize:  $x_0$ , increment ( $\Delta$ ), reduction factor ( $\rho$ ), and termination criterion ( $\epsilon$ ).
2. Set iteration counter ( $k$ ) to 0.
3. Evaluate base point ( $x_k$ ):
  - If movement is successful, update  $x_{k+1}$  and go to Step 5.
  - Else, proceed to Step 4.
4. Check termination:
  - If  $\|\Delta\| < \epsilon$ , terminate.
  - Else, update  $\Delta(i) = \Delta(i)/\rho$  and return to Step 2.
5. Update  $x_{k+1}$  and move to next iteration.
6. Convergence check:
  - If  $f(x_{k+1}) < f(x_k)$ , repeat Step 5.
  - Else, proceed to Step 4.

#### Key Parameters

1. Initial point ( $x_0$ )
2. Increment ( $\Delta$ )
3. Reduction factor ( $\rho$ )
4. Termination criterion ( $\epsilon$ )

The flow-chart for pattern search is given in fig. 2



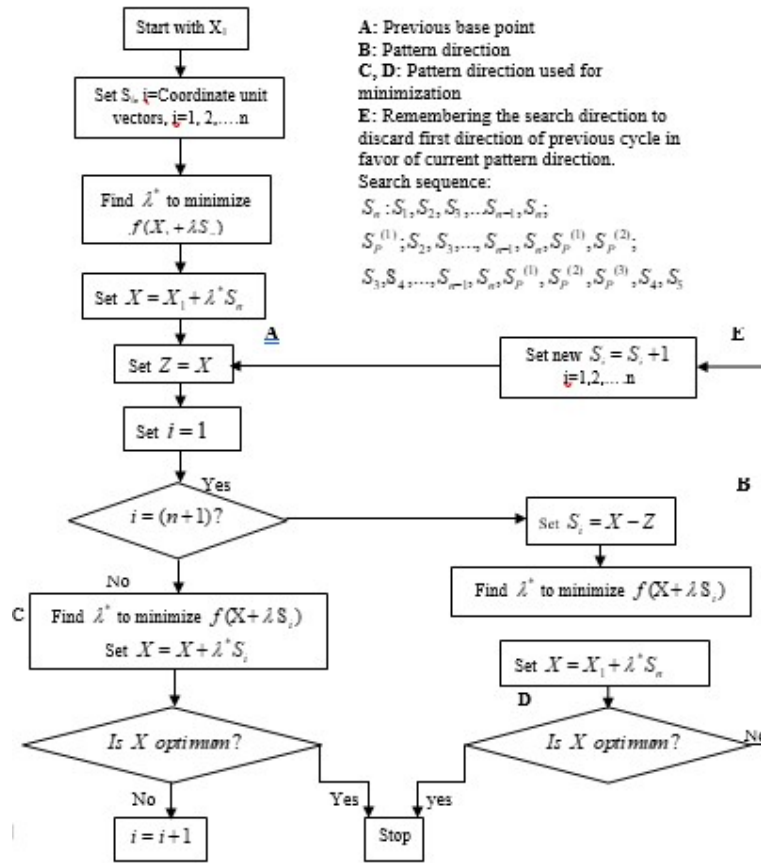


Fig 2: Diagrammatic Representation of Pattern Search through Flow Chart

## 7. CASE-STUDIES

Case-studies on the design problem have been made using three different methods viz. Genetic algorithm, Simulated annealing and pattern search. The results are given below:

### 7.1 Genetic Algorithm

The results obtained from running the program using the genetic algorithm are provided below (Fig. 3 and Table 1).

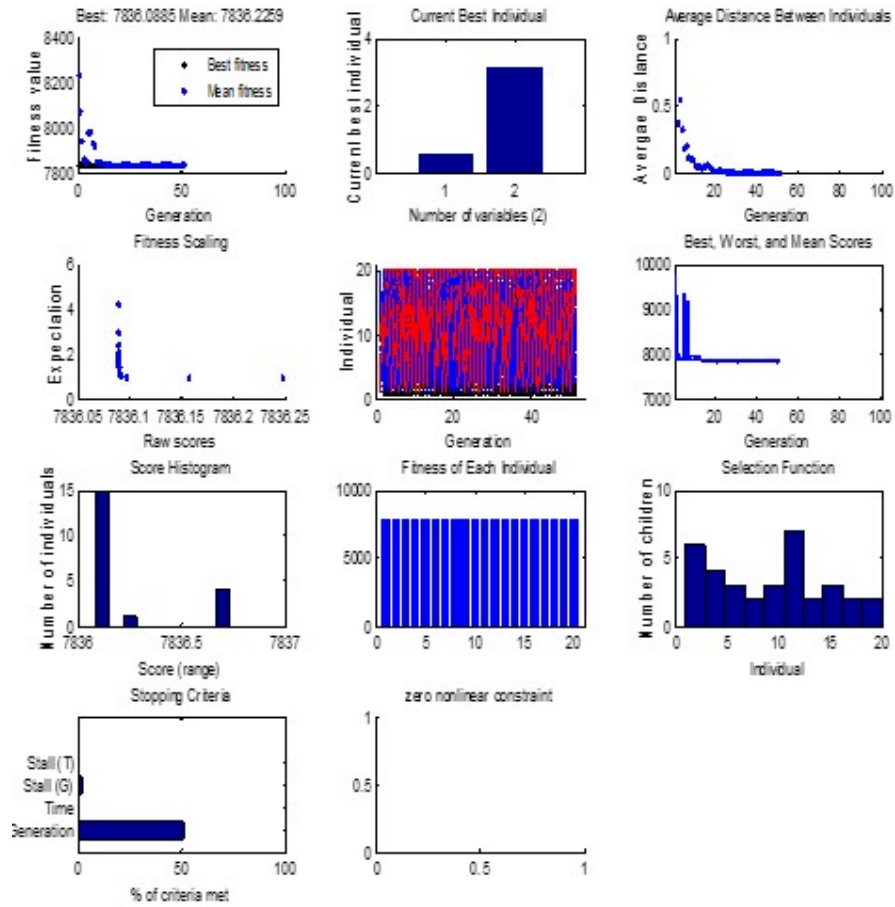


Fig 3: Results using Genetic Algorithm

Generation	f-count	Best f(x)	Variation of mean f(x)	Stall generations
1	40	7850	8030	0
2	60	7841	7910	0
3-13	80-280	7838	7848-8027	0-4
14-21	300-440	7837	7838-7869	0-5
22-51	460-1040	7836	7836-7838	1-4

Table 1: Convergence using GA

## 7.2 Simulated

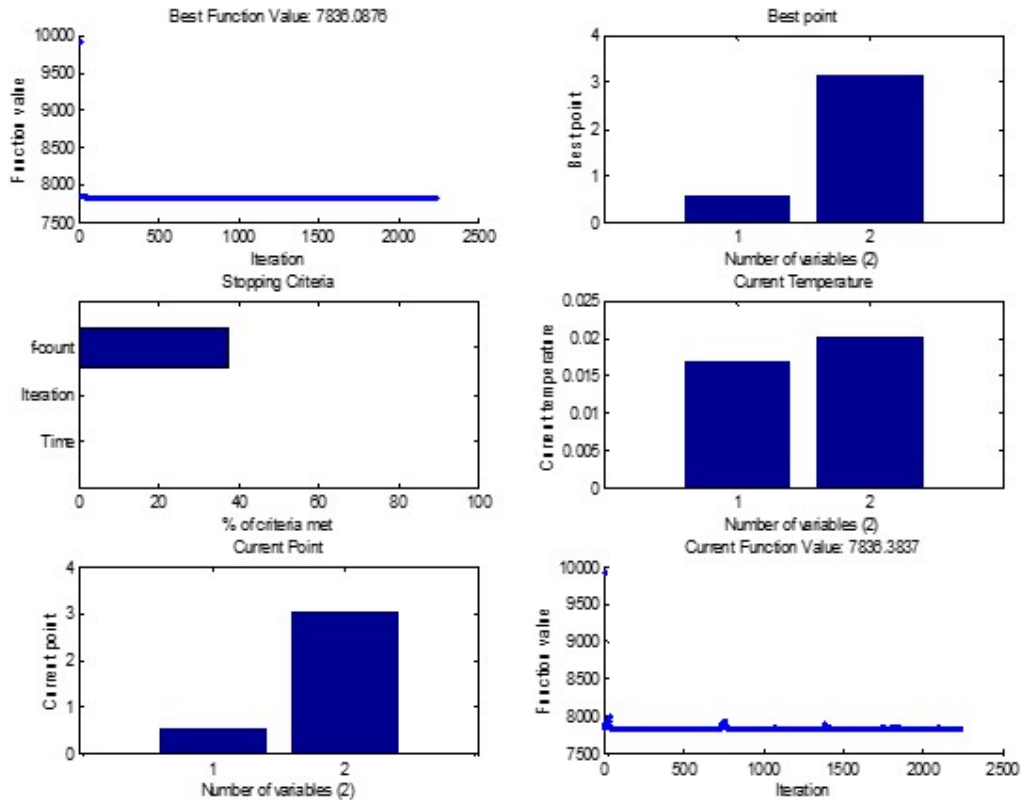


Fig 4: Results Using Simulated Annealing

### Annealing

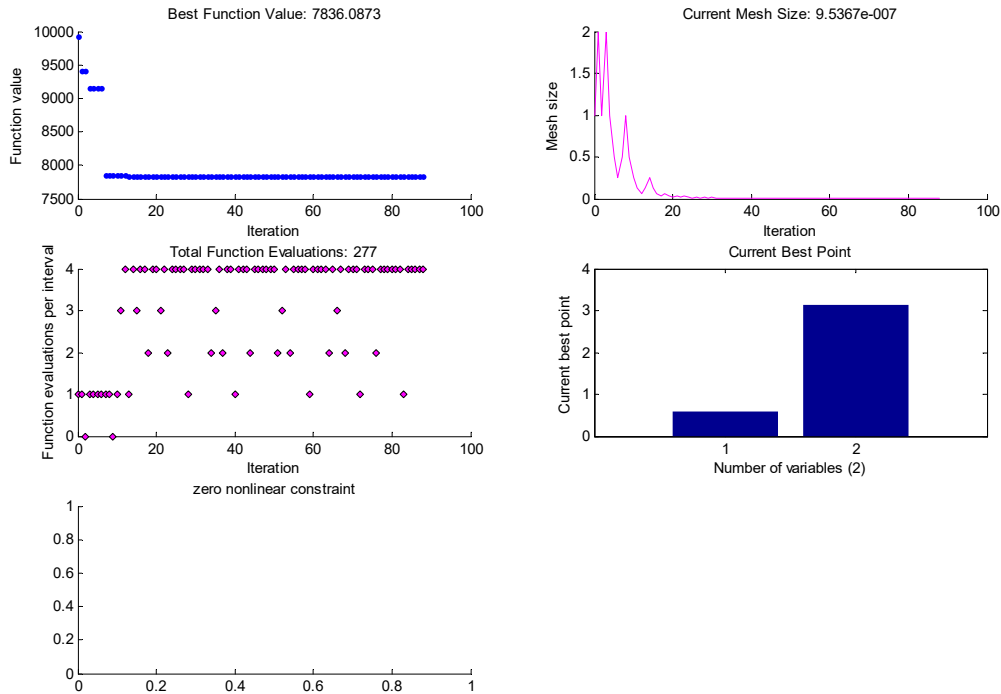
The results generated by running the program with the genetic algorithm are provided in Fig. 4 (above) and table-II (below).

Iteration	f-count	Best f(x)	Current f(x) variation	Mean temperature variation
0	1	9924.45	9924.45	100
10-30	11-31	7845.23	7859.81-7941.28	56.88
40	41	7842.17	7842.17	12.2087
50	51	7837.17	7837.17	7.30977
60-820	61-825	7836.21	7837.71-7836.2	4.3766-3.805e-007
830-890	835-895	7836.19	7836.19-7836.2	0.14636-0.006743
900	905	7836.17	7836.17	0.00403716
910	915	7836.15	7836.16	0.0024172
920	925	7836.14	7836.14	0.00144726
930	935	7836.13	7836.13	0.000866531
940-1200	945-1207	7836.12	7836.12- 7866.07	35.7863-3.073e-6
1210	1217	7836.11	7836.11	0.00795039
1220	1227	7836.1	7836.1	0.00476019
1230-2230	1237-2243	7836.09	7836.09-7867.44	0.00265-9.51e-7

**Table 2: Convergence Using SA**

### 7.3 Pattern Search

The results generated by running the program with pattern search are presented below (Fig. 5 and Table 3).



**Fig. 5 Results Using Pattern Search**

Iteration	f-count	f(x)	Mesh size	Method
0	1	9924.45	1	
1-2	2	9406.85	2	Successful poll/Refine mesh
3-6	3-6	9146.92	2-0.25	Successful poll/Refine mesh
7	7	7854.23	0.5	Successful poll
8-12	8-12	7842.8	0.0625	Successful poll/Refine mesh
13	17	7841.85	0.125	Successful poll
14-17	21-32	7836.72	0.25-0.03125	Successful poll/Refine mesh
18-20	34-42	7836.32	0.0625-0.01563	Successful poll/Refine mesh
21-22	45-49	7836.2	0.03125-0.01563	Successful poll/Refine mesh
23-25	51-59	7836.18	0.03125-0.007813	Successful poll/Refine mesh

26-27	63-67	7836.15	0.01563-0.007813	Successful poll/Refine mesh
28-29	68-72	7836.12	0.01563-0.007813	Successful poll/Refine mesh
30-36	76-79	7836.1	0.01563-0.003906	Successful poll/Refine mesh
37-88	99-277	7836.09	0.007813-9.537e-007	Successful poll/Refine mesh

**Table 3: Convergence using PA**

## 8. CONCLUSION

This study demonstrates the effectiveness of non-classical optimization techniques in minimizing material costs for transformer design. Simulated Annealing emerges as the most efficient method, offering improved results compared to GA and PS. Future research will focus on blending different non-classical techniques to realize a synergistic effect and further enhance optimization outcomes.

This paper aims to optimize the material cost of a small, single-phase dry transformer using three soft computing techniques: Genetic Algorithm, Simulated Annealing, and Pattern Search. The main design variables—emf constant (K) and the window height-to-width ratio (Rw)—were chosen because they directly affect the production cost. For core material, CRGOS was selected, and copper was chosen for the conductor to improve performance. Design constraints were maintained by adjusting the copper current density and iron flux density. The objective was to minimize the combined cost of iron and copper using each optimization method to achieve the lowest production cost. All three methods reached the same solution after a relatively small number of iterations. Using the design variables obtained from the Genetic Algorithm, Simulated Annealing, and Pattern Search, an auxiliary program was employed to calculate the dimensions of the optimized transformer. The cost increased slightly due to rounding real values to the nearest integers where necessary. The performance variables were also calculated, ensuring that all design constraints were met without violation. The paper attempts to argue that the non-classical or probabilistic optimization schemes give a much-improved results compared to the conventional methods like gradient search approach. Moreover a comparative analysis has been carried out amongst different probabilistic optimization schemes and Simulated Annealing has been found to deliver the best result. But this result could be much improved by the application of an elegant blending of different non-classical soft-computing techniques to realize a synergistic effect without applying them individually which is likely to be our future attempt.

## 9. IMPLICATIONS

The results of the study have important implications. They contribute to the optimization of transformer design.

1. Improved material cost reduction.
2. Enhanced efficiency and performance.

### 9.1 Future Research Directions

To further enhance optimization outcomes, future research will focus on:

1. Integrating multiple non-classical techniques to leverage synergistic effects.
2. Exploring advanced soft-computing techniques.
3. Investigating scalability and applicability to larger transformer designs.

## 10. LIMITATIONS

While this study contributes to the existing body of knowledge, it acknowledges the following limitations:

1. Focus on small single-phase dry transformers.
2. Simplified objective function.

### **11. RECOMMENDATIONS**

The study's findings lead to several recommendations. These suggestions are based on the analysis of the data and the results obtained. Implementing these recommendations could improve the overall outcomes. They are intended to guide future actions and decisions in the relevant area.

1. Adopt non-classical optimization techniques for transformer design.
2. Consider Simulated Annealing as a primary optimization method.
3. Explore hybrid approaches combining multiple non-classical techniques.

## **REFERENCES**

- [1] O. W. Andersen, "Optimum design of electrical machines," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-86, no. 6, pp. 707-711, 1967. doi: 10.1109/TPAS.1967.291882
- [2] W. T. Jewell, "Transformer design in the undergraduate power engineering laboratory," *IEEE Transactions on Power Systems*, vol. 5, no. 2, pp. 499-505, 1990. doi: 10.1109/59.54559
- [3] A. Rubaai, "Computer aided instruction of power transformer design in the undergraduate power engineering class," *IEEE Transactions on Power Systems*, vol. 9, no. 3, pp. 1174-1181, 1994. doi: 10.1109/59.336081
- [4] M. Ramamoorthy, *Computer-aided design of electrical equipment*, Affiliated East-West Press Pvt. Ltd., New Delhi, 1987, ISBN 81-85095-57-4.
- [6] A. K. Sawhney, *A Course in Electrical Machine Design*, Dhanpat Rai & Sons, Delhi, 110 001.
- [7] H. M. Rai, *Principles of Electrical Machine Design*, Satya Prakashan, New Delhi, 1985.
- [8] S. S. Rao, *Engineering Optimization – Theory and Practice*, 3rd ed., New Age International (P) Ltd., 1996.
- [9] K. Deb, *Optimization for Engineering Design*, PHI Pvt. Ltd., 1998.
- [10] N. S. Kambo, *Mathematical Programming Techniques*, Revised ed., Affiliated East-West Press Pvt. Ltd., New Delhi, 1991, ISBN 81-85336-47-4.
- [11] D. E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning*, Pearson Education, South Asia, 6th Impression, 2011, ISBN 978-81-775-8829-3.
- [12] E. I. Amoiralis, P. S. Georgilakis, and M. A. Tsili, "Design optimization of distribution transformers based on mixed integer programming methodology," *Journal of Optoelectronics and Advanced Materials*, vol. 10, no. 5, p. 1178, 2008. Available: [https://www.researchgate.net/publication/229148924\\_Design\\_optimization\\_of\\_distribution\\_transformers\\_based\\_on\\_mixed\\_integer\\_programming\\_methodology](https://www.researchgate.net/publication/229148924_Design_optimization_of_distribution_transformers_based_on_mixed_integer_programming_methodology)
- [13] S. Elia, G. Fabbri, E. Nistico, and E. Santini, "Design of cast-resin distribution transformers by means of genetic algorithms," in *International Symposium on Power Electronics, Electrical Drives, Automation and Motion (SPEEDAM 2006)*, Taormina, Italy, May 2006, pp. 1473-1477. doi: 10.1109/SPEEDAM.2006.1650000
- [14] N. Tutkun and A. Moses, "Design optimisation of a typical stripwound toroidal core using Genetic Algorithms," *Journal on Magnetism and Magnetic Materials*, vol. 277, no. 1-2, p. 216, 2004. doi: <https://doi.org/10.1016/j.jmmm.2003.11.002>
- [15] E. I. Amoiralis, P. S. Georgilakis, T. D. Kefalas, M. A. Tsili, and A. G. Kladas, "Artificial intelligence combined with hybrid FEM-BE techniques for global transformer optimization,"

*IEEE Transactions on Magnetics*, vol. 43, no. 4, pp. 1633-1636, 2007. doi: 10.1109/TMAG.2006.892258

[16] L. H. Geromel and C. R. Souza, "The application of intelligent systems in power transformer design," in *IEEE CCECE 2002. Canadian Conference on Electrical and Computer Engineering. Conference Proceedings* (Cat. No. 02CH37373), vol. 1, pp. 285-290, May 2002. doi: 10.1109/CCECE.2002.1015233

[17] R. A. Jabr, "Application of geometric programming to transformer design," *IEEE Transactions on Magnetics*, vol. 41, no. 11, pp. 4261-4269, 2005. doi: 10.1109/TMAG.2005.856921

[18] C. Wu, F. Lee, and R. Davis, "Minimum weight EI core and pot core inductor and transformer designs," *IEEE Transactions on Magnetics*, vol. 16, no. 5, pp. 755-757, 1980. doi: 10.1109/TMAG.1980.1060756

[19] O. W. Andersen, "Optimized design of electric power equipment," *IEEE Computer Applications in Power*, vol. 4, no. 1, pp. 11-15, 1991. doi: 10.1109/67.65030

[20] N. D. Doulamis, A. D. Doulamis, P. S. Georgilakis, S. D. Kollias, and N. D. Hatzargyriou, "A synergetic neural network-genetic scheme for optimal transformer construction," *Integrated Computer-Aided Engineering*, vol. 9, no. 1, pp. 37-56, 2002. doi: 10.3233/ICA-2002-9103

[21] T. H. Pham, S. J. Salon, and S. R. H. Hoole, "Shape optimization of windings for minimum losses," *IEEE Transactions on Magnetics*, vol. 32, no. 5, pp. 4287-4289, 1996. doi: 10.1109/20.538845

[22] N. D. Doulamis and A. D. Doulamis, "Optimal distribution transformers assembly using an adaptable neural network-genetic algorithm scheme," in *IEEE International Conference on Systems, Man and Cybernetics*, vol. 5, pp. 6-pp, Oct. 2002. doi: 10.1109/ICSMC.2002.1176345



## APPENDIX 1: DEVELOPMENT OF THE OBJECTIVE FUNCTION

For the given rating of transformer, EMF/turn =  $K\sqrt{5} = 2.236 K$  volts

Maximum value of flux,  $\phi_m = E_t / (4.44f) = 2.236 K / 222 = 0.01007 K$

Net area of core,  $A_i = \phi_m / B_m = 0.01007 K / 1.4 = 0.00736 K$ ;

Gross area of core,  $A_{gi} = A_i / K_s = 0.0071946 K / 0.93 = 0.00736 K$

For a 3-stepped core, diam. of circumscribing circle,

$$d = \sqrt{A_{gi} / 0.67} = \sqrt{0.00736 K / 0.67} = 0.10745 \sqrt{K}$$

Window area, in  $m^2 = A_w = S / [2.22 f \phi_m K_w \delta \cdot 10^3] = 4.692 \times 10^{-3} / K$

Window width, in m =  $W_w = \sqrt{A_w / R_w} = \sqrt{0.004692 / (K R_w)} = 0.0685 / \sqrt{K R_w}$

Window height, in m =  $H_w = W_w R_w = 0.0685 \sqrt{R_w / K}$

Distance between core centers,  $d_c = d + W_w = 0.10745 \sqrt{K} + 0.0685 / \sqrt{K R_w}$

Length of the largest side of core stamping (for 3-stepped core),  $a = 0.9d = 0.0967 \sqrt{K}$

Overall width,  $W = d_c + a = 0.0685 / \sqrt{K R_w} + 0.2042 \sqrt{K}$

Gross area of yoke is same as gross area of core assuming same flux-density.

The height of yoke, m =  $H_y = A_{gi} / a = 0.00736 K / (0.0967 \sqrt{K}) = 0.08 \sqrt{K}$

Over all height, m =  $H = H_w + 2H_y = 0.0685 \sqrt{R_w / K} + 0.16 \sqrt{K}$

Volume of iron,  $m^3 = V_i = 2(H_w + W)A_i = 2(0.0685(\sqrt{R_w / K} + \sqrt{K R_w}) + 0.2042 \sqrt{K}) \cdot 0.0071946 K$   
 $= 0.0009856(\sqrt{R_w K} + \sqrt{K / R_w}) + 0.002938 K^{1.5}$

Taking density of CRS as 7650 Kg/m<sup>3</sup> and cost of high grade CRS as Rs. 150/- per Kg.

Cost of iron,  $CI = 1131(\sqrt{R_w K} + \sqrt{K / R_w}) + 3371 K^{1.5}$

Mean length of turn, m =  $L_{mt} = \pi(d + W_w / 2) = \pi(0.10745 \sqrt{K} + 0.0685 / 2 / \sqrt{K R_w})$   
 $= 0.3376 \sqrt{K} + 0.1076 / \sqrt{K R_w}$

Total copper area in the window,  $mm^2 =$

$$N_1 I_1 + N_2 I_2 = 2S \times 10^3 / (E_t \delta) = 10000 / (2.236 K \times 2.4) = 1863.4 / K$$

$$\therefore \text{Volume of copper} = 1863.4(0.3376 \sqrt{K} + 0.1076 / \sqrt{K R_w}) / (K \times 10^6)$$

$$= 6.292e-4 / \sqrt{K} + 2.005e-4 / (K^{1.5} \sqrt{R_w})$$

Taking density of copper as 8900 Kg/m<sup>3</sup> and the cost of super enamelled refined copper as Rs. 450/- per Kg,

$$\text{The cost of copper, Rs.} = CC = 8900 \times 450 \times [6.292e-4 / \sqrt{K} + 2.005e-4 / (K^{1.5} \sqrt{R_w})]$$

$$= 2520 / \sqrt{K} + 803 / (K^{1.5} \sqrt{R_w})$$

The Total cost of material (Rs. =  $CM =$

$$1131(\sqrt{R_w K} + \sqrt{K / R_w}) + 3371 K^{1.5} + 2520 / \sqrt{K} + 803 / (K^{1.5} \sqrt{R_w})$$

## APPENDIX 2: DIMENSIONS AND PERFORMANCE VARIABLES OF THE OPTIMIZED TRANSFORMER

The dimensions of the optimized transformer in terms of the optimal values of the design variables and the corresponding performance variables are given below:

K & R found out by soft-computing techniques: SA, PS, & GA

Chosen values of emf constant,  $K = 0.576$ ;

Chosen values of window height/width ratio,  $R_w = 3.14$

Chosen core material: CRGOS. Stacking factor,  $K_s = 0.93$ ; Cost of iron/Kg = 150/-

Chosen flux-density,  $B_m = 1.4$  wb/m<sup>2</sup>; Iron loss/Kg at this flux-density = 1.331 W

Chosen conductor material: COPPER;  
Chosen current density=  $2.4 \text{ A/mm}^2$   
Cost of copper/Kg= Rs. 450/-  
Resistivity of copper at operating temperature =  $0.022 \text{ } \Omega/\text{m/mm}^2$   
EMF/Turn= 1.288V  
Net/ Gross area of iron=  $4.1574\text{E-}03 \text{ m}^2 / 4.4704\text{E-}03 \text{ m}^2$   
3-stepped core has been used.  
Diameter of the core circle=  $8.168\text{E-}02 \text{ m}$   
Length of the largest side of stamping =  $0.074 \text{ m}$   
Area of window=  $8.0616\text{E-}03 \text{ m}^2$   
Width, height of window:  $0.051 \text{ m} / 0.16 \text{ m}$   
Distance between core center=  $0.13268 \text{ m}$   
Total width of core=  $0.20668 \text{ m}$   
Width and height of yoke:  $0.074\text{m}; 0.06 \text{ m}$   
Total height of core,  $H = 0.28 \text{ m}$   
Volume/weight of iron=  $3.0489\text{E-}03 \text{ m}^3 / 23.324 \text{ Kg}$   
Total cost of iron = Rs.3499/-  
Number of turns of the primary/secondary= 178/ 89  
Primary/Secondary current=  $21.739\text{A} / 43.478 \text{ A}$   
Mean length of turn=  $0.33677 \text{ m}$   
Volume/weight of copper=  $1.086\text{E-}03 \text{ m}^3 / 9.665 \text{ Kg}$   
Cost of copper= Rs.4349/-  
Total cost of material= Rs.7848/-  
Iron loss in W, % iron loss:  $31.046 ; 0.62093$   
Copper loss in W, % copper loss:  $137.61 ; 2.7523$   
Efficiency at full load, 0.8 lagging p.f.=  $0.9595$   
Magnetising current w.r.t primary =  $0.31053 \text{ A}$   
% Magnetising current= 1.4285  
% Core loss current=  $0.6209$ ; % no load current= 1.558  
Magnetising current w.r.t primary=  $0.3105 \text{ A}$   
% Magnetising current= 1.428  
% Core loss current=  $0.6209$   
% No load current= 1.5576  
Leakage reactance w.r.t. primary=  $0.4957 \text{ } \Omega$   
% Leakage reactance = 4.6854  
% Voltage regulation at full load, 0.85 lagging p.f.= 4.808  
Temperature rise of winding at full load =  $42^\circ \text{ C}$